Optional methods of measuring air flow in rectangular ducts produce different results

Equal Area vs. Log-Tchebycheff

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In November 1997, I was hired to design a HVAC system and to perform the final test and balance (TAB) of all systems for a $7-million chemical laboratory facility in Angarsk, Siberia. After I carefully set the flow of air in a rectangular duct at 19,000 cfm, Russian ventilation specialists told me that they measured it at 21,000 cfm.

Such a discrepancy was typical for rectangular ducts on that project. Although I was able to convince the technicians that my readings were accurate, I was curious enough about their procedures that I built a full-scale fan and duct assembly (see photograph) to conduct experimental duct traversals upon my return home. What I discovered was that the Russians did not use the equal-area method, which is popular in the United States, as I had assumed (a description of the Russians' method is beyond the scope of this article). But even more surprising — and of greater importance to design engineers and TAB contractors in the United States — was my discovery that the equal-area method is in itself flawed, failing to consider lower "fall-off" air velocities along duct walls and, thus, consistently overstating air flows. As I began to develop my own traversal method to capture the slower velocities, I learned of a seldom-used procedure called log-Tchebycheff.

Through my tests, I discovered that results from the equal-area method always are in error — from 5 percent to a consistent 9 percent up to 20 percent above actual air flow — which supports statements made in ANSI/ASHRAE Standard 111-88[1] and the ASHRAE '97 Fundamental Handbook.[2] Yet, despite this and many TAB contractors' acknowledgment that there is little or no difference in contract cost, labor, and time between the equal-area and log-Tchebycheff methods, the former is used almost exclusively in the United States, with the differences in results routinely dismissed as "insignificant."

This article describes and compares the equal-area and log-Tchebycheff methods for rectangular ducts and shows why the latter is superior. Also, it makes recommendations for widening the use of the log-Tchebycheff method throughout the industry. The objective is correct testing and balancing of HVAC designs, with the ultimate goal of proper air distribution, air pressures, and indoor air quality (IAQ) in all buildings.

By specifying that the log-Tchebycheff method be used for TAB reports, design engineers would reduce the number of disputes they often find themselves in because of inaccurate test results.

PHOTO A (above). The full-scale fan and duct assembly, which features a 48-by-12-in. rectangular duct. Note the red plugs, which are stopping holes drilled at 3, 9, 15, 21, 27, 33, 39, and 45 in. across the top and 3 and 9 in. down the side for the traversal of the duct using the equal-area method.

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[1] *Superscript numerals indicate references listed at end of article.*

BACKGROUND

The equal-area method of measuring air flow in rectangular ducts and the log-linear method of measuring air flow in round ducts were devised by a group of engineers sometime before 1952. Later, during the 1970s and '80s, two organizations — the Associated Air Balance Council (AABC) and the National Environmental Balancing Bureau (NEBB) — were established, becoming the only nationally recognized agencies that field-test and certify air-flow measurements for rectangular and round ducts.

In 1977, a mathematician with the last name of Tchebycheff (pronounced "che-boo-chef") developed his own methods of measuring air flow in rectangular ducts and round ducts. Simply called "log-Tchebycheff," the methods were published in International Organization for Standardization No. 39663.[3]

About 10 years later, the AABC and the NEBB adopted both of Tchebycheff's methods and included them in their standards as an option to the equal-area method for rectangular ducts and the log-linear method for round ducts.[4,5] However, no contracted AABC or NEBB company has used or will use the log-Tchebycheff method for rectangular ducts. (The log-Tchebycheff and log-linear methods for round ducts produce the same results.)

TRAVERSING A DUCT

In traversing a duct, holes are cut down one side or across the top or bottom of the duct at spacings determined by the method being used. A measurement probe, such as a hot-wire anemometer or pitot tube, then is inserted in the first hole.

The dimension of the duct walls and the method being used determine the depths to which the probe is inserted and the number of air-velocity measurements — in feet per minute (fpm) — that are taken. For example, relative to the inner wall of a 12-in. duct, readings would be taken at 3 and 9 in. using the equal-area method and at 1.5, 6, and 10.5 in. using the log-Tchebycheff method. (The larger the size of the duct, the greater the number of traversal points, regardless of method.) The process is repeated for each hole, with the readings taken at all traversal points then added, averaged, and multiplied by the duct's cross-sectional area to determine volumetric air flow, measured in cubic feet per minute (cfm). Figures 1 and 2 show the traversal points for a 30-in. square duct using the log-Tchebycheff and equal-area methods, respectively.

Note that the algorithm that determines the spacing of traversal points for the equal-area method results in equal distances between points, while the algorithm for the log-Tchebycheff method results in a higher density of readings toward the middle of the duct, as well as readings closer to the walls. This better approximates the shape of an air stream.

Although the log-Tchebycheff method usually requires the drilling of an additional hole and the taking of additional measurements, the extra work should require little or no additional cost.

COMPARING THE METHODS

My experiments showed that the equal-area method overstates air flow, which can be attributed to the measurement and averaging of only the air velocities of the interior. The log-Tchebycheff method, on the other hand, takes into account lower air velocities — caused by friction and other phenomena — along duct walls and in duct corners, which are averaged with higher air velocities of the interior.

To illustrate the potential threat to the safe operation of HVAC systems posed by use of the equal-area method, let's say that the method is used to measure air flow in a rectangular duct at the design flow rate of 60,000 cfm. While the log-Tchebycheff method would have produced a more accurate measurement of 55,000 cfm, the 8.3-percent difference is "insignificant," falling within the industry standard of tolerance of ±10 percent. However, let's now say that the air flow in that same duct is set, using

Votes of confidence

The following exclusively endorse the log-Tchebycheff method for rectangular ducts:

- **AMCA Publication 201-85,** *Fans and Systems.*
- **AMCA Publication 501-93,** *Application Manual for Air Louvers.*
- **AMCA Publication 502-89,** *Damper Application Manual.*
- **AMCA Publication 503-93,** *Fire, Ceiling and Smoke Dampers Applications.*
- **AMCA Standard 500-D-98,** *Laboratory Methods of Testing Dampers for Rating.*
- **AMCA Standard 500-L-99,** *Laboratory Methods of Testing Louvers for Rating.*
- **AMCA Standard 803-87,** *Site Performance Test Standard.*
- **ANSI/AMCA Standard 210-85,** *Laboratory Methods of Testing Fans for Aerodynamic Performance.*
- **ANSI/AMCA Standard 610-93,** *Methods of Testing Airflow Measurement Stations for Rating.*
- **ANSI/ASHRAE Standard 51-1985,** *Laboratory Methods of Testing Fans for Rating.*
- **ANSI/ASHRAE Standard 111-88,** *Practices for Measurement, Testing, Adjusting and Balancing of Building Heating, Ventilation, Air Conditioning and Refrigeration Systems.*
- **ASHRAE '97 Fundamental Handbook,** *Measurement and Instruments.*
- **ASHRAE '99 Applications Handbook,** *Testing, Adjusting and Balancing.*
- **HEVAC 1981,** *Fan Application Guide*, HVAC Manufacturing Association.
- **International Organization for Standardization 3966,** *Measurement of Fluid Flow in Closed Conduits.*

Log-Tchebycheff method for rectangular ducts

SPACING ALGORITHM

Duct wall dimension (in.)	Traverse points per side	Position relative to interior wall (fraction of total)						
< 30	5 (a)	0.074	0.288	0.500	0.712	0.926	—	—
30-36	6	0.061	0.235	0.437	0.563	0.765	0.939	—
> 36	7	0.053	0.203	0.366	0.500	0.634	0.797	0.947

CALCULATED POSITIONS

Dimension (in.)	No. Spacings	Position relative to interior wall (in.) 1	2	3	4	5	6	7	8
6	3(a)	1.0	3.0	5.0					
7	3(a)	1.0	3.5	6.0					
8	3(a)	1.0	4.0	7.0					
9	3(a)	1.0	4.5	8.0					
10	3(a)	1.5	5.0	8.5					
11	3(a)	1.5	5.5	9.5					
12	3(a)	1.5	6.0	10.5					
13	3(a)	1.5	6.5	11.5					
14	3(a)	1.5	7.0	12.5					
15	3(a)	1.5	7.5	13.5					
16	4(a)	1.5	5.8	10.2	14.5				
17	4(a)	1.5	6.2	10.8	15.5				
18	4(a)	1.5	6.5	11.5	16.5				
19	4(a)	1.5	6.8	12.2	17.5				
20	5	1.5	5.8	10.0	14.2	18.5			
21	5	1.5	6.0	10.5	14.9	19.5			
22	5	1.6	6.3	11.0	15.7	20.4			
24	5	1.8	6.9	12.0	17.1	22.2			
26	5	1.9	7.5	13.0	18.5	24.1			
27	5	2.0	7.8	13.5	19.2	25.0			
28	5	2.1	8.0	14.0	19.9	25.9			
29	5	2.1	8.4	14.5	20.6	26.9			
30	6	1.8	7.0	13.1	16.9	22.9	28.2		
32	6	1.9	7.5	14.0	18.0	24.5	30.1		
34	6	2.1	8.0	14.8	19.1	26.0	31.9		
35	6	2.1	8.2	15.3	19.7	26.8	32.9		
36	6	2.2	8.5	15.7	20.3	27.5	33.8		
37	7	2.0	7.5	13.5	18.5	23.5	29.5	35.0	
38	7	2.0	7.7	13.9	19.0	24.1	30.3	36.0	
39	7	2.1	7.9	14.3	19.5	24.7	31.1	36.9	
40	7	2.1	8.1	14.6	20.0	25.4	31.9	37.9	
42	7	2.2	8.5	15.4	21.0	26.6	33.5	39.8	
44	7	2.3	8.9	16.1	22.0	27.9	35.1	41.7	
46	7	2.4	9.3	16.8	23.0	29.2	36.7	43.6	
48	7	2.5	9.7	17.6	24.0	30.4	38.2	45.5	
50	7	2.6	10.1	18.3	25.0	31.7	39.8	47.4	
52	7	2.8	10.6	19.0	26.0	32.9	41.4	49.2	
54	7	2.9	10.9	19.8	27.0	34.2	43.0	51.1	
56	7	3.0	11.4	20.5	28.0	35.5	44.6	53.0	
58	7	3.1	11.8	21.2	29.0	36.8	46.2	54.9	
60	7	3.2	12.2	21.9	30.0	38.0	47.8	56.8	

Notes:

(a) Through results of fan/duct assembly experiments, it is acceptable to apply these variations:

Dimension	No. spacings	Positions
< 9 in.	3	First two points 1.0 in. from opposite walls; third point in the middle.
10-15 in.	3	First two points 1.5 in. from opposite walls; third point in the middle.
16-19 in.	4	First two points 1.5 in. from opposite walls; other two points spaced equally between first two points.

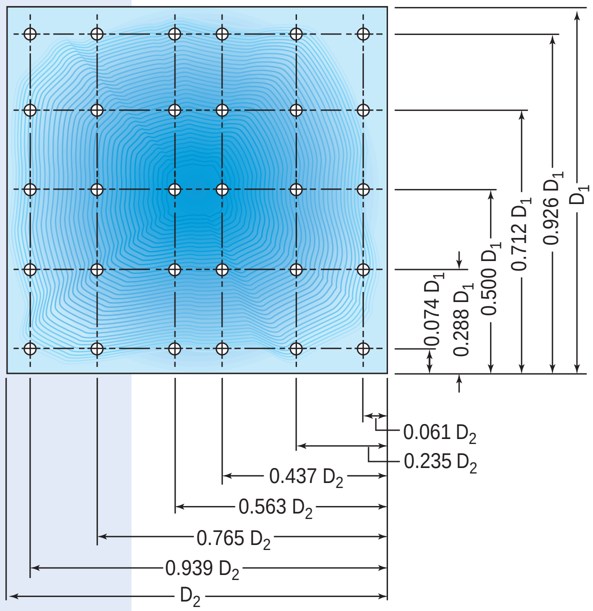


FIGURE 1. Log-Tchebycheff traversal points and spacing algorithm for 6-in. square duct.

the equal-area method as a gauge, at 55,000 cfm. Little would the contractor know that the air flow actually is around 50,000 cfm, which would have been discovered using the log-Tchebycheff method and which would be unacceptable by industry standards at 16.7 percent below design flow. Eventually, the building would experience serious IAQ problems.

The following analogy may help clarify the issue: You are driving your car on a freeway where the minimum speed is 60 mph and the maximum speed is 80 mph. Preferring to drive in the slower right-hand lane at 60 mph, you are unaware that your speedometer is miscalibrated by -5 mph. A police officer clocks you at 55 mph (-8.3 percent), a speed not quite slow enough to warrant a ticket. Still believing that you are driving 60 mph, you decide to slow down to an acceptable 55 mph. The police officer then clocks you at 50 mph (-16.7 percent) and pulls you over. In effect, your speedometer represents the equal-area method and the radar gun represents the log-Tchebycheff method.

FIELD EXPERIENCE

As an example of how inaccurate TAB reporting can be, consider the following. In 1998, I designed a con-

Equal-area method for rectangular ducts

SPACING ALGORITHM (see diagram)

x	y (a)	z/2	z + z/2	z(2) + z/2	z(3)+ z/2	z(4) + z/2	z(5) + z/2	z(6) + z/2	z(7) + z/2	z(8) + z/2

CALCULATED SPACINGS

Dimension (in.)	No. Spacings	Position relative to interior wall (in.) 1	2	3	4	5	6	7	8	9
6	1	3.0								
7	1	3.5								
8	2	2.0	4.0							
9	2	2.3	6.7							
10	2	2.5	7.5							
11	2	2.7	8.3							
12	2	3.0	9.0							
13	3	2.2	6.5	10.8						
14	3	2.3	7.0	11.7						
15	3	2.5	7.5	12.5						
16	3	2.7	8.0	13.3						
17	3	2.8	8.5	14.2						
18	3	3.0	9.0	15.0						
19	4	2.4	7.1	11.9	16.6					
20	4	2.5	7.5	12.5	17.5					
21	4	2.6	7.9	13.1	18.4					
22	4	2.8	8.2	13.7	19.2					
24	4	3.0	9.0	15.0	21.0					
26	5	2.6	7.8	13.0	18.2	23.4				
27	5	2.7	8.1	13.5	18.9	24.3				
28	5	2.8	8.4	14.0	19.6	25.2				
29	5	2.9	8.7	14.5	20.3	26.1				
30	5	3.0	9.0	15.0	21.0	27.0				
32	6	2.7	8.0	13.3	18.7	24.0	29.3			
34	6	2.8	8.5	14.2	19.8	25.5	31.2			
35	6	2.9	8.8	14.6	20.4	26.2	32.1			
36	6	3.0	9.0	15.0	21.0	27.0	33.0			
37	7	2.6	7.9	13.2	18.5	23.8	29.0	34.4		
38	7	2.7	8.1	13.6	19.0	24.4	29.9	35.3		
39	7	2.8	8.3	13.9	19.5	25.1	30.6	36.2		
40	7	2.9	8.6	14.3	20.0	25.7	31.4	37.1		
42	7	3.0	9.0	15.0	21.0	27.0	33.0	39.0		
44	8	2.8	8.2	13.7	19.3	24.7	30.2	35.7	41.2	
46	8	2.9	8.6	14.4	20.1	25.8	31.6	37.3	43.1	
48	8	3.0	9.0	15.0	21.0	27.0	33.0	39.0	45.0	
50	9	2.8	8.3	13.9	19.4	25.0	30.5	36.1	41.6	47.2
52	9	2.9	8.7	14.4	20.2	26.0	31.8	37.5	43.3	49.1
54	9	3.0	9.0	15.0	21.0	27.0	33.0	39.0	45.0	51.0
56	8(c)	3.5	10.5	17.5	24.5	31.5	38.5	45.5	52.5	
58	8(c)	3.6	10.8	18.1	25.4	32.6	39.8	47.1	54.4	
60	8(c)	3.8	11.2	18.7	26.3	33.7	41.2	48.7	56.2	

Notes:
(a) y (the least number of points) = x in. (duct-wall size) / (any whole number or fraction between 1.0 and 6.0 in.)
(b) z in. (hole spacings) = x in. (duct-wall size) / y. For duct sizes equal to 56 in. and larger, use only 8 points.

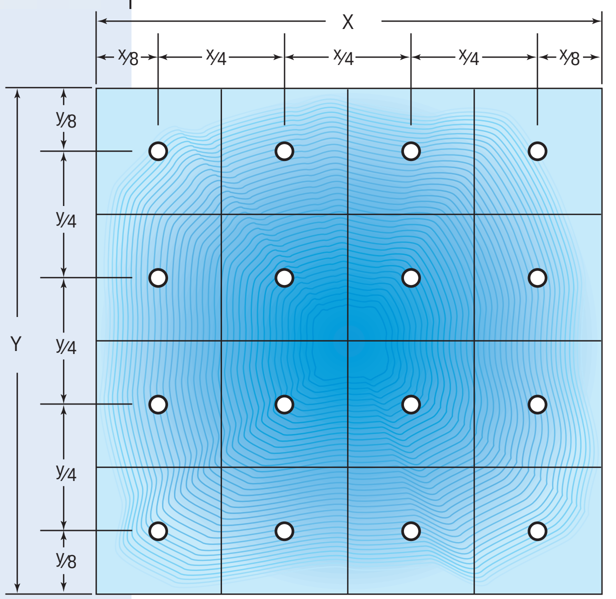


FIGURE 2. Equal-area traversal points and spacing algorithm for 6-in. square duct.

ventional decoupled air-conditioning system for a new two-story elementary school. While reading the certified TAB report, I discovered that:

- Although an air-handling unit designed for 5830 cfm was supplying a very good 104 percent of design outside-air flow directly to the space, the main duct *was traversed by the equal-area method to be 125 percent.*
- Another air-handling unit, designed for 3800 cfm, was supplying a very good 107 percent of design outside-air flow directly to the space, but the main duct *was traversed by the equal-area method to be 119 percent.*
- Although an exhaust fan designed for 2875 cfm was exhausting a very good 98 percent of design air flow directly from the space, the main duct *was traversed by the equal-area method to be 143 percent.*
- Another exhaust fan, designed for 2295 cfm, was exhausting a very good 99 percent of design air flow directly from the space, but the main duct *was traversed by the equal-area method to be 156 percent.*

During construction, these same duct systems were pressure-tested and certified by that same TAB contractor as not exceeding 1.5-percent leakage. If he had traversed the main duct first and set the air flow to what he believed to have been the design quantity, my

Fan/Duct Assembly Experiments

The sketch below shows a full-scale belt-driven fan and duct assembly that was built for the performance of duct-traversal experiments. Results from the experiments, which were conducted on three separate days, show slight variations due to normal ambient variables such as outside air temperature and density. Supply air from the fan was traversed in the round duct that leads to the rectangular duct. Air in the rectangular duct then was traversed using equal-area method spacings, with extra measurements taken at the top and bottom of the duct. Pitot tube readings taken horizontally were averaged for each vertical spacing. The extra measurements enabled the creation of a complete vertical profile of air-flow velocity in the duct (see the table of average test velocities and the illustration of the vertical profile). The readings taken for the 3-in. and 9-in. vertical spacings correspond precisely to those called for by the equal-area method (see Figure 2). The resulting curves then were used to derive readings corresponding to log-Tchebycheff spacings (see Graphical Analysis Results) as per Figure 1.

Fan/duct assembly and traversal parameters

DUCT DIMENSIONS

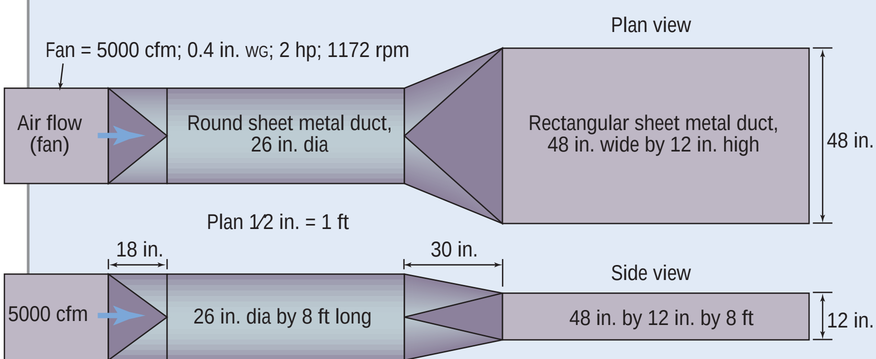
Width: 48 in.
Height: 12 in.
Cross-sectional area: 4 sq ft

HORIZONTAL TRAVERSAL SPACINGS (in. from wall)

Equal-area	3.0	9.0	15.0	21.0	27.0	33.0	39.0	45.0
Log-Tchebycheff	2.5	9.7	17.6	24.0	30.4	38.2	45.5	

VERTICAL TRAVERSAL SPACINGS (in. from bottom)

Equal-area	3.0	9.0			
Log-Tchebycheff	0.074	0.288	0.500	0.712	0.926



Average pitot tube measurements*

Height (in.)	Day 1 (fpm)	Day 2 (fpm)	Day 3 (fpm)
0.0	1135	1099	1019
3.0	1387	1428	1384
9.0	1460	1473	1389
12.0	982	1074	950

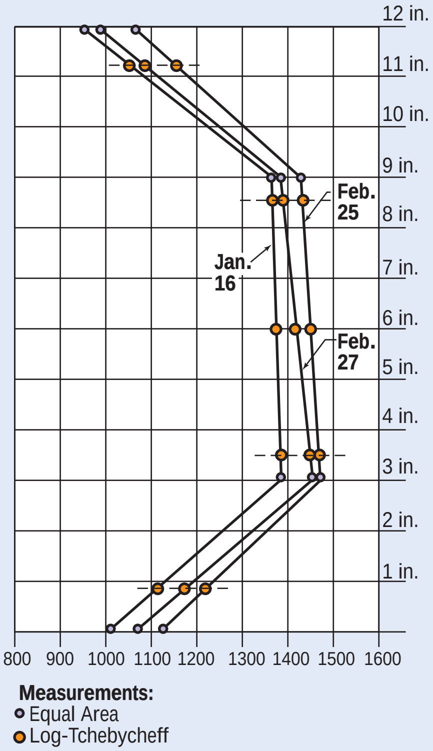
well-designed system would have become short of air flow within the spaces, which would have led to a serious IAQ problem.

Later, I reviewed TAB reports for five previous projects that certified that air flows were within the industry standard of ±10 percent when they were, in fact, in excess of -10 percent of design. Had the log-Tchebycheff method been used, accurate air-flow results would have been produced.

OUTCOMES OF INACCURATE TESTING

The following are some typical problems that will occur when the air flow in a rectangular duct is measured and set using the equal-area method:

- Systems that are certified field-tested and balanced will operate with less air flow than recorded.
- In hospitals, quantities of operational supply air and outside air for operating rooms and other areas always will be less than specified and be in violation of code, affecting space-to-space pressure relationships.
- Operational solvent ventilation systems will be less than specified and be in violation of code that specifies a minimum of one exhaust air change every five minutes.
- The operational energy efficiency ratio of cooling equipment and the coefficient of performance of heating/cooling equipment will be less than specified and, consequently, be in violation of the energy code.
- The operational annual fuel utilization efficiency of oil- and gas-fired burners that require mechanically supplied combustion air will be less than specified and be in violation of the energy code. Also, insufficient combustion air could cause incomplete combustion and depressurization, leading to the formation of carbon monoxide (CO) backdrafting and the presence of CO inside of the building.
- Manual dampers in ducted systems will need excessive throttling, which wastes energy.
- Leakage in metal and glass-fiber ducts will be reported incorrectly at an unacceptable level of up to 25 percent.
- Fan efficiency will be less than specified and be in violation of the energy code.



Average traversal results
(equal-area)

Height (in.)	Day 1 (fpm)	Day 2 (fpm)	Day 3 (fpm)
3.0	1387	1428	1384
9.0	1460	1473	1389
Average	*1424*	*1451*	*1387*

Graphical analysis results
(log-Tchebycheff)

Height (in.)	Day 1 (fpm)	Day 2 (fpm)	Day 3 (fpm)
0.074	1060	1090	1170
0.288	1370	1390	1430
0.500	1380	1420	1450
0.712	1380	1450	1470
0.926	1120	1180	1230
Average	*1262*	*1306*	*1350*

** Measurements taken along horizontal dimension are averaged to give vertical profile.*

Comparisons
(percent difference)

Log-Tchebycheff compared to equal-area, i.e., (LT-EA)/LT	-8.7
Equal-area compared to entry flow rate, i.e., (EA-RD)/RD	7.0
Log-Tchebycheff compared to round duct flow rate, i.e., (LT-RD)/RD	-1.6

Calculations

	Average flow rate			
	Day 1 (cfm)	Day 2 (cfm)	Day 3 (cfm)	Avg. (cfm)
Equal-area	5694	5802	5546	5681
Log-Tchebycheff	5048	5224	5400	5224
Round-duct flow*	5157	5424	5351	5311

** Round-duct flow rate measured by traversing 26-in. dia supply duct from 5000-cfm fan.*

- Operational smoke evacuation exhaust systems will be less than specified and be in violation of applicable code by more than the minimum 10-min air change.
- Toilet exhaust systems will be less than specified and be in violation of applicable code.
- Grease exhaust systems will be less than the 1500 fpm specified and be in violation of applicable code.
- Outdoor air-ventilation make-up air will be less than specified and be in violation of applicable code. This will cause the building to become depressurized, with unconditioned air pulled into occupied spaces.

CONCLUSIONS

A fact not easily accepted by my engineering friends is that, today, the most important professional involved in a HVAC project is the AABC- or NEBB-certified TAB contractor. A perfectly designed and constructed system will not work until his or her work is done correctly.

Since my firm began specifying in construction documents that the log-Tchebycheff method be used for rectangular ducts, there have been no disputes or claims for additional cost during bidding or after a contract was awarded. And since that time, I have not had post-construction air-flow distribution problems, air-pressure problems, or air-noise problems on any of 80 projects.

RECOMMENDATIONS

For the correct method of measuring air flow in rectangular ducts to be utilized, certified testing agencies would have to change what they have been doing since 1952. Because, I have been told, agencies in the United States would be reluctant to change voluntarily, fire, energy, and mechanical codes would have to be amended, specifying log-Tchebycheff as the only method for measuring air flow in rectangular ducts or making reference to ANSI/ASHRAE Standard 111-88 or the ASHRAE '97 Fundamental Handbook, both of which state that the equal-area method is with error and that the log-Tchebycheff method for rectangular ducts produces correct results.

I am a mechanical design engineer registered in 18 states who prepares HVAC construction documents for schools, medical facilities, office buildings, laboratories, and other buildings. I have no interest in earning a living in the HVAC test-and-balance business. I simply want to avoid the misleading and incorrect TAB reports I have received in the past. **HPAC**

REFERENCES

1) ANSI/ASHRAE Standard 111-88, *Practices for Measurement, Testing, Adjusting and Balancing of Building Heating, Ventilation, Air Conditioning and Refrigeration Systems*, American Society of Heating, Refrigerating and Air-Conditioning Engineers, Atlanta, Ga., 1988.

2) *ASHRAE. 1997, ASHRAE Handbook-Fundamentals*. Atlanta: The American Society of Heating, Refrigerating and Air-Conditioning Engineers Inc.

3) International Organization for Standardization No. 3966, *Measurement of Fluid Flow in Closed Conduits.*

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5) *NEBB. 1998, Procedural Standards for Testing, Adjusting, and Balancing of Environmental Systems*. Rockville, Md.: National Environmental Balancing Bureau.