EQUAL AREA vs. LOG-TCHEBYCHEFF

Revisited

In a study initiated by *HPAC Engineering*, methods of measuring air flow in rectangular ducts are put to the test

EDITOR'S NOTE: In December 1999, HPAC Engineering *published "Equal Area vs. Log-Tchebycheff," an article by Ernest L. MacFerran, PE, championing the little-known Log-Tchebycheff method of measuring air flow in rectangular ducts, which, the author claimed, produces more-accurate results than does the widely used Equal Area method. The article generated much response from readers. Some vowed always to specify the "Log-T" method for test-and-balance reports, while others dismissed the differences in accuracy as insignificant. In an effort to further the discussion,* HPAC Engineering *asked the Iowa Energy Center to test the two methods. The results are presented here.*

Marty Pieper of Systems Management and Balancing Inc. measures duct velocities at Traverse Plane No. 1. Note the difference in measurement-point locations between the Equal Area (top) and Log-Tchebycheff methods.

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Pitot-tube traverses commonly are used during test-and-balance procedures to determine volumetric air-flow rates in ductwork. For rectangular ducts, there are two accepted methods of determining the grid of locations where measurements should be taken, namely, the Log-Tchebycheff method adopted by the American Society of Heating, Refrigerating and Air-Conditioning Engineers (ASHRAE)[1,2] and the Equal Area method supported by the Associated Air Balance Council (AABC).[3] Both methods determine duct air velocity by sampling velocity pressure at individual points in the traverse plane. Where they differ is in the rules that prescribe the location of those points. The Log-Tchebycheff method purports greater accuracy because the loca-

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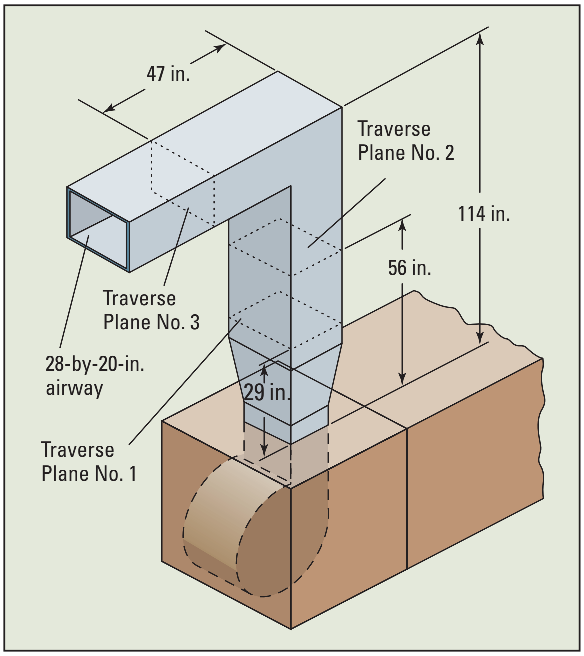


FIGURE 1. Schematic of ductwork and traverse-plane locations.

tion of its points accounts for friction loss at the duct walls.[1]

This article compares air-flow rates obtained with the Log-Tchebycheff and Equal Area methods and examines the influence traverse-plane location had on the measurements. Testing was conducted at the Iowa Energy Center's Energy Resource Station (ERS), which supports two commercial-scale air-handling systems serving matched pairs of test rooms and one general-service system serving the remainder of the building. The testing was part of an effort to identify duct-velocity profiles and calibrate air-flow-measuring stations for the general-service air-handling system. The tests were intended to provide a comparison of the traverse methods under the less-than-ideal flow conditions frequently encountered in the field. The testing was limited to one main-supply-duct size and a specific set of operating conditions.

TEST CHARACTERISTICS

Ductwork. Figure 1 is a schematic of the air-handling-system supply-air ductwork. The air-handling-unit upblast discharge provides air directly to this main supply-air-ductwork section. The dimensions of the ductwork go from the 21 in. by 18 in. of the air-handling-unit outlet to the 30 in. by 22 in. of the sheet-metal duct, where the measurements were taken. A 1-in. liner reduces the duct's interior dimensions to 28 in. by 20 in. Although not shown in the diagram, turning vanes are installed in the 90-degree elbow.

The locations of the three traverse planes (a traverse plane is located at the tip of a Pitot-tube probe) are shown in Figure 1. For the velocities anticipated, 100-percent effective duct length corresponding to a uniform velocity profile would be expected at two-and-one-half equivalent duct diameters downstream from the fan outlet.

The general-service air-handling system used for the tests. It serves the classrooms, offices, and common areas of the Energy Resource Station with a nominal capacity of 7,800 cfm.

The three traverse planes can be summarized as follows:

- System effect and the effect of a fan discharge are represented at Traverse Plane No. 1, which is approximately 50-percent effective duct length from the outlet of the fan.
- Traverse Plane No. 2 is located approximately 100-percent effective duct length from the outlet of the fan.
- The duct elbow with turning vanes introduces an upstream disturbance for Traverse Plane No. 3 at a distance slightly greater than one equivalent duct diameter. Approximately 32 in. downstream from Traverse Plane No. 3 is the first branch duct takeoff from the main supply duct.

Measurement grids. For a duct with a 28-by-20-in. airway, the Log-Tcheby-

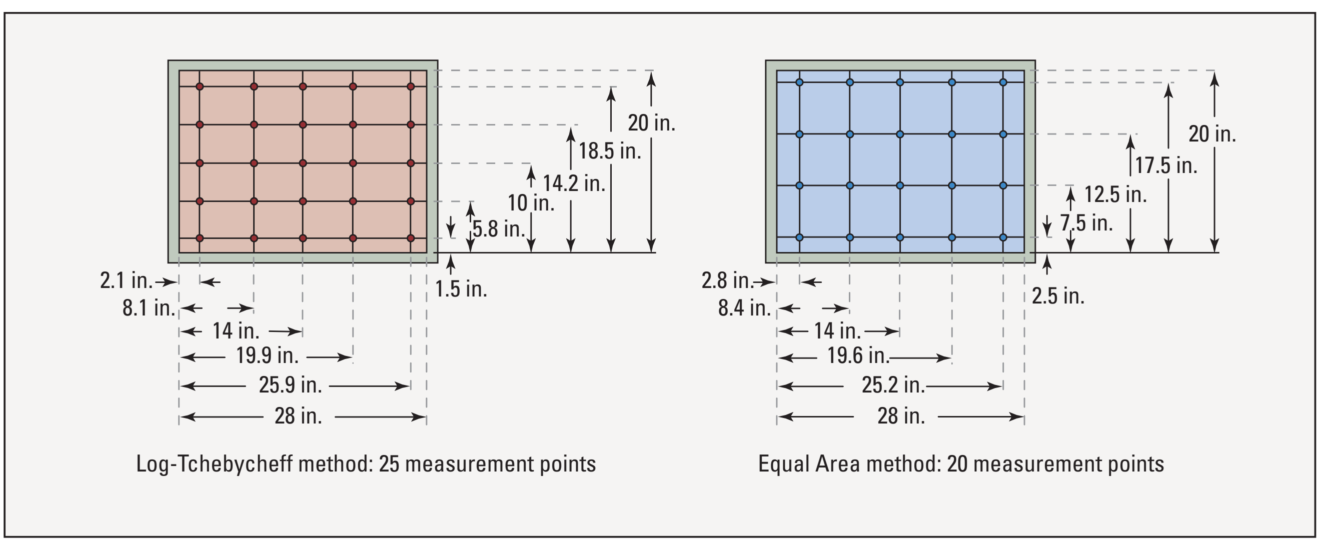


FIGURE 2. Log-Tchebycheff and Equal Area traverse grids for a 28-by-20-in. airway.

cheff method calls for a five-by-five grid of unequally spaced measurements,[1] while the Equal Area method requires a five-by-four grid[3] with the distance between measurements no more than 6 in. The locations of the measurement points for both methods are shown in Figure 2.

Air-handling-system operation. Prior to and throughout the test period, the general-service air-handling system was operated in a steady-state, constant-volume mode. The supply and return fans were overridden to fixed-speed operation, and

A floor-up view of the supply-air ductwork, showing the direction of air flow from the fan discharge. Traverse Plane Nos. 1 and 2 are in the vertical section of the ductwork, while Traverse Plane No. 3 is in the horizontal section.

the outside-, return-, and exhaust-air dampers were positioned for 100-percent return air. The fan-powered, variable-air-volume box dampers were fixed at the full open position, with the fans disabled. To determine the stability of system operation, an electronic flow-measuring-station signal was recorded each minute. The system maintained a stable air-flow rate, with a peak-to-peak range consistently less than 2.3 percent of the mean flow.

Performing measurements. The measurements were performed using a Shortridge Airdata Multimeter Model ADM-860 with a Certificate of Recalibration dated seven weeks prior to the tests. This instrument provides automatic pressure compensation to account for non-standard conditions. Attaching a temperature probe to the instrument provides temperature compensation.

The instrument was operated in a differential-pressure mode, with velocity computed internally in units of feet per minute (fpm). Using the calibration data sheet, the uncertainty of the velocity measurements was estimated to be ±3 percent of the reading.

To minimize measurement error resulting from instrument operation, the services of a testing-and-balancing engineer were enlisted. Well-qualified with 17 years of field experience, Marty Pieper of Systems Management and Balancing Inc. performed all of the measurements reported in this article.

Data sets. Measurements were made at each of the traverse planes shown in Figure 1 using both the Log-Tchebycheff and the Equal Area measurement locations. At each location, three measurements of air velocity were obtained consecutively and then averaged to establish a mean velocity for that location. The entire procedure was repeated to produce 12 data sets based on accepted standards defined by ASHRAE and AABC.

It was determined that the most uniform velocity profile was located in the horizontal section of duct at Traverse Plane No. 3. Ideally, the reference air-flow rate would have been established by measuring differential pressure across a primary instrument, such as a flow nozzle. For this experiment, such a measurement was not practical. Instead, the reference air-flow rate was determined using a Pitot-tube traverse of a much higher resolution. In particular, measurements were taken with a 14-by-10 grid, with the Pitot tube positioned at the center of 2-by-2-in. squares. For the reference case, only a single measurement was taken at each location.

RESULTS

Results of all of the tests are presented in Table 1, with velocity profiles for the shaded cases plotted in figures 3-5. Both ASHRAE and AABC provide guidelines regarding the acceptability of velocity profiles. These guidelines say that for a velocity distribution to be acceptable, 75 percent or more of the velocity measurements must be greater than 1/10 of the maximum velocity

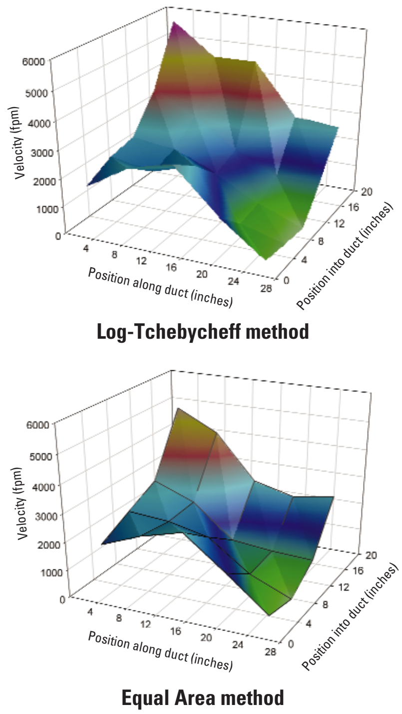
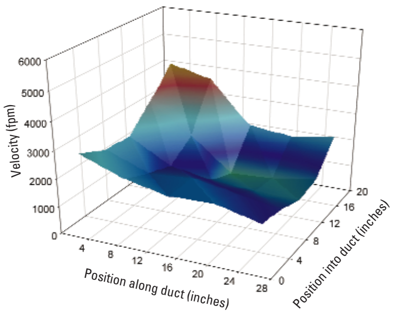


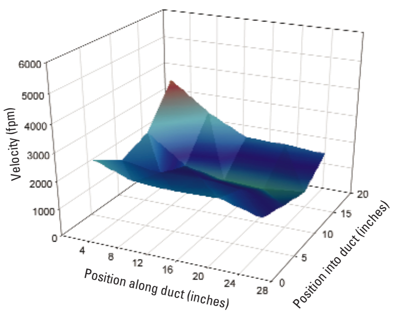
FIGURE 3. Velocity profiles obtained at Traverse Plane No. 1, Run No. 2.

of that profile. The ASHRAE guideline further states that for a distribution to be considered ideal, 80 to 90 percent of the velocity measurements must be greater than 1/10 of the maximum velocity of that profile. At Traverse Plane No. 1, 80 to 90 percent of the velocity measurements were greater than 1/10 of the maximum velocity, while at both of the other traverse planes, 100 percent of the velocity measurements were greater. By the above criteria, then, all of the profiles recorded at all three traverse locations satisfy the requirement for an ideal distribution.

The velocity profiles obtained with the Log-Tchebycheff and Equal Area methods at Traverse Plane No. 1 are presented in Figure 3. Although the profiles are very non-uniform, they are consistent between the two methods. Velocities on the far left side at the back of the duct (position along the duct close to 0 in. and position into the duct approaching 20 in.) approach or exceed 5,000 fpm, while velocities in the front right corner (position along the duct close to 28 in. and position into the duct approaching 0 in.) are very low. In fact, velocities at some locations in the front right corner are negative with both methods and were recorded as zero in accordance with the ASHRAE standard.[1]



Log-Tchebycheff method



Equal Area method

FIGURE 4. Velocity profiles obtained at Traverse Plane No. 2, Run No. 2.

The non-uniformity of the profiles at Traverse Plane No. 1 was expected given the abrupt transition disturbance just upstream. The highest velocities occurred at a location directly in line with the fan discharge, while the lowest velocities occurred at a location directly in line with the most severe transition. The air-flow rates at Traverse Plane No. 1 showed a wide variation both between the two methods and between the two runs performed with each method.

Figure 4 shows the velocity profiles obtained with the Log-Tchebycheff and Equal Area methods at Traverse Plane No. 2. Although, as with Traverse Plane No. 1, the profiles are very similar, the range of velocities is substantially smaller. The profiles are interesting in that they have the appearance of an inverted "D." Instead of the highest velocities being at the center of the duct, as is the case with fully developed turbulent flow in straight ducts, the highest velocities are near the walls.

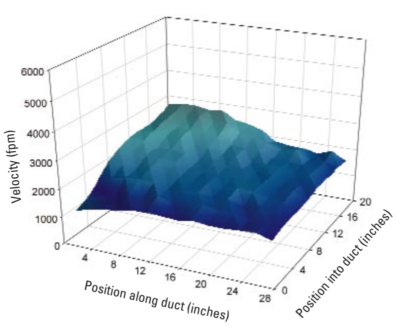


FIGURE 5. High-resolution Equal-Area-method velocity profile obtained at Traverse Plane No. 3.

		LOG-TCHEBYCHEFF		EQUAL AREA	
Traverse Plane No.	Run No.	Air-flow rate (cfm)	Relative error (%)[a]	Air-flow rate (cfm)	Relative error (%)[a]
1	1	7,811	-0.04	7,288	-6.73
	2	8,204	4.99	7,623	-2.44
2	1	7,620	-2.48	7,352	-5.91
	2	7,639	-2.24	7,187	-8.02
3	1	7,700	-1.46	7,838	0.31
	2	7,740	-0.95	7,843	0.37
3	Reference[b]			7,814	

[a] The relative error is determined from:

$$\frac{Q\text{–Reference}}{\text{Reference}} \times 100\%$$

where:
Q = Equal-Area- or Log-Tchebycheff-method air-flow rate.

[b] The reference air-flow rate was obtained using a 14-by-10 grid. All other Equal-Area-method results were obtained with a five-by-four grid, while all Log-Tchebycheff-method results were obtained with a five-by-five grid. Recommended grids for both methods are dependent on duct size.

TABLE 1. Results of the air-flow measurements.

Table 1 shows that while the air-flow rates obtained with both methods at Traverse Plane No. 2 are less than the reference value of 7,814 cfm, the rates obtained with the Log-Tchebycheff method are more consistent between the two runs (7,620 cfm and 7,639 cfm) and are within 2.5 percent of the reference value.

The high-resolution Equal Area profile obtained at Traverse Plane No. 3 is shown in Figure 5. This profile, obtained with a grid of 140 measurement points, shows that the velocities, although still not displaying the classic "D" shape, are much more uniform. Because the profiles obtained with the Log-Tchebycheff method (five-by-five grid) and the Equal Area method (five-by-four grid) also were highly uniform, they are not presented.

Table 1 shows that the two air-flow rates obtained with the Log-Tchebycheff method at Traverse Plane No. 3 differ from one another by only 40 cfm and differ from the reference value by less than 1.5 percent, while the two air-flow rates obtained with the Equal Area method are nearly the same and differ from the reference value by less than 0.4 percent. Even though the Log-Tchebycheff measurements slightly underpredict the reference value, and the Equal Area measurements slightly overpredict it, both are very satisfactory. In fact, the differences in the results obtained with the two methods and those obtained with the high-resolution Equal Area grid are well within the estimated uncertainty of the velocity measurements. The implication is that, with the results from Traverse Plane No. 3, no conclusion can be made regarding which method is more accurate.

CONCLUSIONS

The primary conclusion that can be drawn from these tests is that the uniformity of the velocity profile offered by the traverse-plane location has a more significant influence on an air-flow

The vertical portion of the main supply-air ductwork of the general-service air-handling system. The yellow duct plugs identify Traverse Plane No. 1.

measurement than does the method (Log-Tchebycheff or Equal Area) used to determine the measurement grid.

At Traverse Plane No. 3, where the velocity profiles are very uniform, the Log-Tchebycheff and Equal Area methods produce results that are in excellent agreement with the reference air-flow rate determined using a high-resolution grid traverse. At Traverse Plane No. 2, the velocity profiles are less uniform, with the average measurement of the Log-Tchebycheff method approximately 2.4-percent less than the reference value and the average measurement of the Equal Area method approximately 7-percent less than the reference value. At this location, the additional measurement points of the Log-Tchebycheff method provide the resolution necessary to capture the velocity profile. At Traverse Plane No. 1, the velocity profiles are the least uniform, and the results are the least consistent. This is the only location at which negative readings were obtained, a factor that may have contributed to the inconsistency of the measurements.

The variances identified at traverse planes 1 and 2 occur under velocity-distribution conditions considered ideal by the criterion that 80 to 90 percent of the velocity measurements be greater than 1⁄10 of the maximum velocity. This reinforces the importance of this criterion in determining acceptable velocity profiles for the traverse-plane location selected. Improved confidence in the measured values is expected as the 1⁄10 threshold increases.

The testing reported here considers only a single duct size and air-flow rate; therefore, it is not possible to draw any conclusions about the generality of the results. The results do, however, suggest that additional research aimed at comparing the accuracy of the Log-Tchebycheff and Equal Area methods is merited. In particular, the scope of the comparisons should be extended to consider a range of air-flow rates, duct sizes, and configurations, with measurements taken under field conditions.

ACKNOWLEDGMENT

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