The following relationship can be derived from the ideal gas laws by dividing the ideal gas equation for water vapor by the ideal gas equation for air.

$$\frac{p_H}{p - p_H} = w \times \frac{R_H}{R_a}$$

Where:

p_H = The partial pressure of water vapor in the sample in lb/ft^2 absolute.

p = The total pressure of the air and water vapor mixture in the sample in lb/ft^2 absolute.

w = The humidity ratio (humidity ratio is sometimes called specific humidity) in pounds of water per pound of dry air. If it is given in grains per pound, you can convert it to pounds by using the conversion factor of 7,000 grains per pound.

R_H = The gas constant for water vapor (85.8 ft. per°F).

R_a = The gas constant for air (53.3 ft. per°F).

The equations can also be arranged in the following form.

$$w = \frac{R_a}{R_H} \times \frac{p_H}{p - p_H}$$

A special case for saturation.

$$w_s = \frac{R_a}{R_H} \times \frac{p_s}{p - p_s}$$

Where:

p_s = The partial pressure of water vapor in the sample at saturation in lb/ft^2 absolute.

You can also determine the dew point if you know the humidity ratio and the pressure by solving the relationship for the partial pressure of water at saturation and then using a steam table to look up the saturation temperature associated with the partial pressure of water at saturation.

$$w_s = \frac{R_a}{R_H} \times \frac{p_s}{p - p_s}$$

Therefore:

$$w_s \times \frac{R_H}{R_a} = \frac{p_s}{p - p_s}$$

$$\left(w_s \times \frac{R_H}{R_a}\right) \times (p - p_s) = p_s$$

$$p \times \left(w_s \times \frac{R_H}{R_a}\right) - p_s \times \left(w_s \times \frac{R_H}{R_a}\right) = p_s$$

$$p \times \left(w_s \times \frac{R_H}{R_a}\right) = p_s + \left[p_s \times \left(w_s \times \frac{R_H}{R_a}\right)\right]$$

$$p \times \left(w_s \times \frac{R_H}{R_a}\right) = p_s \times \left[1 + \left(w_s \times \frac{R_H}{R_a}\right)\right]$$